derived by assuming that the resonances are those of the single rings. This leads to the resonance frequencies

$$f_1 = \frac{n \cdot c_0}{2\pi r_{mo}(\epsilon_{eff})^{1/2}} \quad f_2 = \frac{m \cdot c_0}{2\pi r_{mi}(\epsilon_{eff})^{1/2}}, \qquad m,n = 1,2,\cdots \quad (5)$$

where ϵ_{eff} is the effective dielectric constant of the single microstrip line of the same width w as the resonator has. As the field distributions in Fig. 3 show, (5) should be at least a first approximation for the resonance frequencies, because the resonances are mainly those of the single rings. Fig. 4 shows that (5) is quite a good approximation, especially the growing difference between f_1 and f_2; with growing distance, s is described quite well. Up to the fifth higher order resonance ($n = 5$, $m = 5$), the accuracy of (5) is better than 3 percent for all resonators which have been examined (0.05 cm $\leq s \leq$ 0.5 cm, Polyguide material, $\epsilon_r = 2.32$, $h = 0.156$ cm), whereas the agreement between (4) and the experimental results is not so good (accuracy of about 5 percent for f_1 and about 9 percent for f_2, $m = 4$, $n = 4$, and $s = 0.5$ cm).

As has been shown in Section II, the unrolled double-ring resonator is the straight double-line resonator of different line lengths as shown in Fig. 1. So the $n \cdot \lambda_g$-resonance frequencies of the straight double-line resonator should be an approximation for the resonance frequencies of the double-ring resonator. Fig. 5 shows the comparison between the $n \cdot \lambda_g$-resonance frequencies of the straight double-line resonator and the measured resonance frequencies of the double-ring resonators. The agreement between theory and experiment is excellent for all values of s, as long as the mode numbers n,m are small ($n,m \leq 3$). The deviation between theory and experiment increases with the increasing value of n,m. This is due to the fact that with larger n,m (meaning with increasing resonance frequencies) the difference Δl of the circumferences of the two rings becomes of the order $\lambda_g/2$, which leads to a bad approximation of the double-ring resonator by the straight double-line resonator.

In conclusion, as far as we think, the double-ring resonator principally is not a good arrangement to measure the phase velocities of the even and the odd modes of a coupled microstrip line. Only in the case of very closely coupled lines can it be used to measure v_{phe} and v_{pho}, for in this case all three described theories are good approximations for the resonance frequencies and, e.g., (4) can be used to measure ϵ_{effe} and ϵ_{effo}. Furthermore the mean circumference of the resonator in this case should be larger than $5\lambda_g$ to avoid the influence of the curvature of the lines on the resonance frequencies (see, e.g , [5]).

REFERENCES

[1] J. W. Gould and E. C. Talboys, ' Even- and odd-mode guide wavelengths of coupled lines in microstrip," *Electron. Lett.*, vol. 8, pp. 121–122, Mar. 1972.

[2] W. J. Getsinger, "Dispersion of parallel-coupled microstrip," *IEEE Trans. Microwave Theory Tech.* (Short Papers), vol. MTT-21, pp. 144–145, Mar. 1973.

[3] I. Wolff, "The measurement of the effective dielectric constant of the even and the odd mode on coupled microstrip lines," *Nachrichtentech. Z.* (Germany), vol. 27, pp. 30–34, Jan. 1974

[4] ——, "Microstrip bandpass filter using degenerate modes of a microstrip ring resonator," *Electron. Lett.*, vol. 8, pp. 302–303, June 1972.

[5] I. Wolff and N. Knoppik, "The microstrip ring resonator and dispersion measurement on microstrip lines," *Electron. Lett.*, vol. 7, pp. 779–781, Dec. 1971.

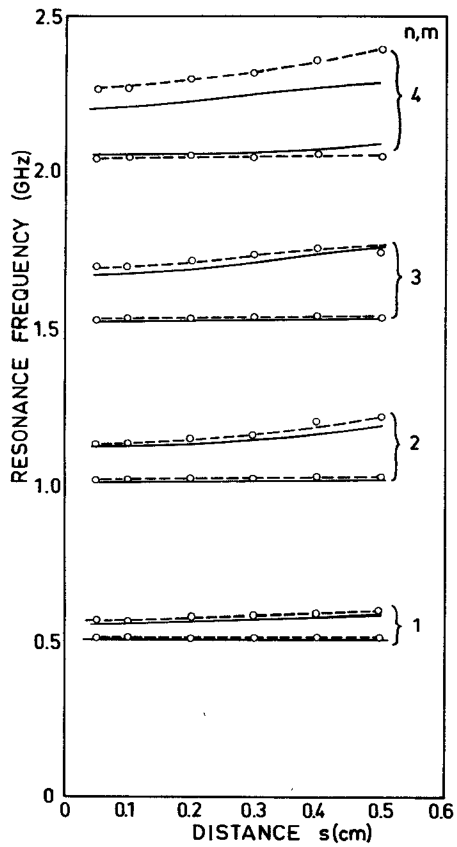

Fig. 5. Resonance frequencies of the double-ring resonator as a function of the distance s between the rings. ——— calculated by (3), ○ experimental results. Resonator dimensions as in Fig. 4.

A Coupled-Line Model for Dispersion in Parallel-Coupled Microstrips

HERBERT J. CARLIN, FELLOW, IEEE, AND PIER P. CIVALLERI, MEMBER, IEEE

***Abstract*—A new circuit model is derived for parallel-coupled microstrip consisting of two separate pairs of coupled lines. Each pair consists of a homogenoeus TEM line coupled to a homogeneous TE line. One pair represents the hybrid even mode, the other represents the odd mode. Data calculated from the model are compared with experimental dispersion data for various parallel-coupled microstrip geometries. Agreement is excellent.**

The procedure for deriving the equivalent circuit is an example of a general technique for using coupled lines to model longitudinally uniform but transversely inhomogeneous lossless waveguide.

The representation of fields in longitudinally uniform but transversely inhomogeneous metallic-bound waveguides by the use of an infinite number of coupled TE and TM transmission lines was first introduced by Schelkunoff [1]. More recently, it was shown that by appropriately truncating the Schelkunoff representation, one can obtain practical models consisting of a finite number of coupled lines, from which the propagation functions of the structure can be approximated [2]. Moreover, even in cases in which the Schelkunoff parameters cannot be easily calculated, a practical coupled-line model can

Manuscript received August 5, 1974; revised November 13, 1974. This work was supported in part by NSF Grant GK-21012X.

H. J. Carlin is with the School of Electrical Engineering, Cornell University, Ithaca, N.Y. 14850. He was a Visiting Professor at the Istituto di Elettrotecnico, Genoa, Italy.

P. P. Civalleri is with the Istituto Elettrotecnico Nazionale Galileo Ferraris, Turin, Italy.

still be constructed on the basis of appropriate physical intuition and suitable use of experimental data. For example, a pure TEM coupled to a TE line was recently successfully used as a model to describe dispersion in a microstrip [3].

In this short paper, we present a coupled-line model for a parallel-coupled microstrip. The model has a clear physical basis, is conceptually very simple, can be readily generalized by the addition of more coupled lines in the equivalent circuit to approximate higher order modes, and stems from a basic technique which can be applied to a wide variety of propagating systems. The model is shown in Fig. 1. Each of two TEM transmission lines representing the low-frequency even mode (index e) and the low-frequency odd mode (index o), respectively, is coupled with a different TE line; each pair of TE–TEM coupled lines is separate, i.e., uncoupled to the other pair.

To justify the assumption of this model at the outset, we can make the following remarks.

1) The microstrip can be thought of as enclosed in a rectangular metallic box; or a shielded enclosure with large dimensions compared to dielectric thickness may actually be present. The dimensions of the box are actually parameters of the approximation and do not necessarily correspond to a physical shield.

2) In a transversely *homogeneous* longitudinally uniform metallic-bound structure, all TE, TM, and TEM modes are uncoupled. In this particular problem, the TM mode is not involved.

3) Since the microstrip is magnetically homogeneous, it is reasonable to assume that all modes are still magnetically uncoupled.

4) Electric coupling in the microstrip will take place mostly between the fundamental TEM modes and the lowest higher modes which, in the empty rectangular waveguide, are just the TE_{10} and the TE_{01}, the former with the even TEM, the latter with the odd TEM (see Fig. 1).

These points are obviously of a heuristic nature: the validity of the model will be demonstrated by comparison with experimental results.

The series-impedance network and the shunt-admittance network per unit length of both pairs of coupled lines are represented by the following matrices:

$$Z(p) = \left|\begin{array}{cc|cc} \mu_o h_e & 0 & 0 & 0 \\ 0 & \mu_o & 0 & 0 \\ \hline 0 & 0 & \mu_o h_o & 0 \\ 0 & 0 & 0 & \mu_o \end{array}\right| p \qquad (1)$$

$$Y(p) = \left[\begin{array}{cc|cc} \dfrac{\bar{\epsilon}_e\epsilon_o}{h_e} & -k_e\dfrac{\bar{\epsilon}_e\epsilon_o}{(h_e)^{1/2}} & 0 & 0 \\ -k_e\dfrac{\bar{\epsilon}_e\epsilon_o}{(h_e)^{1/2}} & \bar{\epsilon}_e\epsilon_o & 0 & 0 \\ \hline 0 & 0 & \dfrac{\bar{\epsilon}_o\epsilon_o}{h_o} & -k_o\dfrac{\bar{\epsilon}_o\epsilon_o}{(h_o)^{1/2}} \\ 0 & 0 & -k_o\dfrac{\bar{\epsilon}_o\epsilon_o}{(h_o)^{1/2}} & \bar{\epsilon}_o\epsilon_o \end{array}\right] p + \left[\begin{array}{cc|cc} 0 & 0 & 0 & 0 \\ 0 & K_{ce}^2/\mu_o & 0 & 0 \\ \hline 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & K_{co}^2/\mu_o \end{array}\right] p^{-1}. \qquad (2)$$

In (1) and (2), $p = \sigma + j\omega$ is the complex frequency; μ_o and ϵ_o are the constitutive constants of free space; $\bar{\epsilon}_e$ and $\bar{\epsilon}_o$ are the effective static dielectric constants of the even and the odd modes, respectively; K_{ce} and K_{co} are the cutoff wavenumbers of the uncoupled TE_{10} and TE_{01} lines, respectively. h_e and h_0 are dimensional parameters taking into account the microstrip transverse section geometry; k_e and k_o are the capacitive coupling coefficients. The per-unit length circuits shown in Fig. 1 correspond to (1) and (2).

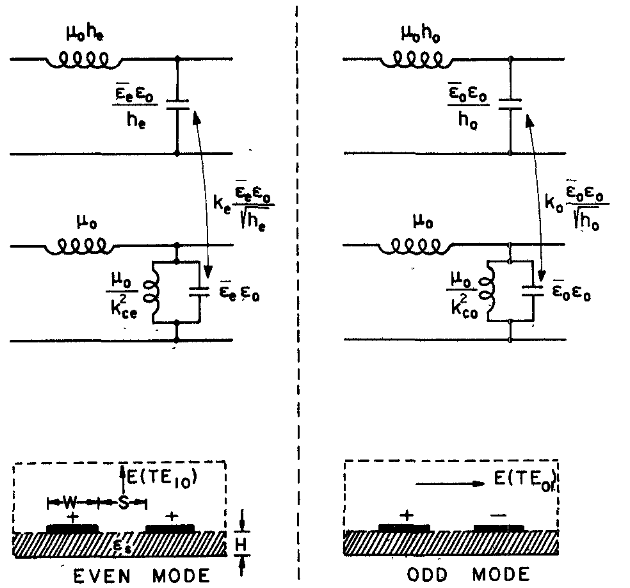

Fig. 1. Coupled-line model for parallel-coupled microstrips, showing even and odd coupling networks.

By solving the secular equation

$$\det [Z(p)Y(p) - \gamma^2 I_4] = 0$$

where I_4 is the four-rowed identity matrix, and taking into account the definition of the effective dielectric constant

$$\epsilon_{\text{eff}}(p) = \gamma^2(p)/\epsilon_o\mu_o p^2$$

one obtains for the previous quantity the following expression:

$$\epsilon_{\text{eff}}^{i}(p)_{1,2} = \bar{\epsilon}_i + (K_{ci}^2 v_0^2/2p^2) \pm [(K_{ci}^4 v_0^4/4p^4) + k_i^2\bar{\epsilon}_i^2]^{1/2} \qquad (3)$$

where v_0 is the velocity of propagation of electromagnetic waves in free space; index i can take the values e or o, depending on whether the mode is even or odd; and indices 1 and 2 refer to the choice of the plus or minus sign, respectively, for the square root. The plus sign corresponds to the modes that propagate down to dc. The other pair of modes with the minus sign (index 2) are cut off at a finite frequency. Equation (3) has the same form already found for a single microstrip [3], and this suggests that the same techniques can be used to determine the unknown constants k_e, k_o, K_{ce}, and K_{co}.

The coupling coefficients k_e and k_o are determined from the condition that at infinite frequency all of the wave energy is concentrated in the substrate of relative dielectric constant ϵ_s. Thus, taking the limit of (3), $p \to \infty$, i.e., $\epsilon_{\text{eff}}^{i}(\infty)_1 = \epsilon_s$,

$$k_i = (\epsilon_s - \bar{\epsilon}_i)/\bar{\epsilon}_i. \qquad (4)$$

The cutoff wavenumbers are determined by the semiempirical formula given in [3]

$$K_{ci}^2 = (k_i/R)[(2\pi)^2/12G_iH^2]\bar{\epsilon}_i(Z_{ai}/376.7)^2$$

where

$$R = [2(7)^{1/2} - 1]^{1/2}/6 \qquad (6)$$

$$G_i = 0.500 + 0.001\, Z_{ai}^{3/2} \qquad (7)$$

and $Z_{ae} = Z_{0e}/2$, $Z_{ao} = 2Z_{0o}$, where Z_{0e} and Z_{0o} are the even and odd static characteristic impedances of the coupled microstrips [4]. The dielectric substrate thickness is H.

The previous considerations were used to obtain circuit models of the microstrip structures whose dispersion was measured and reported by Gould and Talboys [5]. The results comparing the experimental data of [5] and the dispersion calculated from our circuit model are shown in Fig. 2. For each set of microstrip dimensions labeled by a number, letters a and b refer to the dispersive odd and even TEM [1] modes, respectively, for that geometry as calculated from (3) using the plus sign, and with $p = j\omega$. The geometric and static data for the specific microstrips considered, taken from [4] and [5], are presented in Table I. The cutoff modes which are paired with the dispersive TEM modes are not shown in Fig. 2 since they are strongly attenuated over the frequency band shown; their cutoff frequencies are easily calculated from (3) as

$$f_{ci} = [K_{ci}v_0/2\pi(1 - k_i^2)^{1/2}\bar{\epsilon}_i^{1/2}]. \tag{8}$$

Since the parameters of the approximation are based on the fundamental mode, this equation should probably be used with some caution. It has not been experimentally verified.

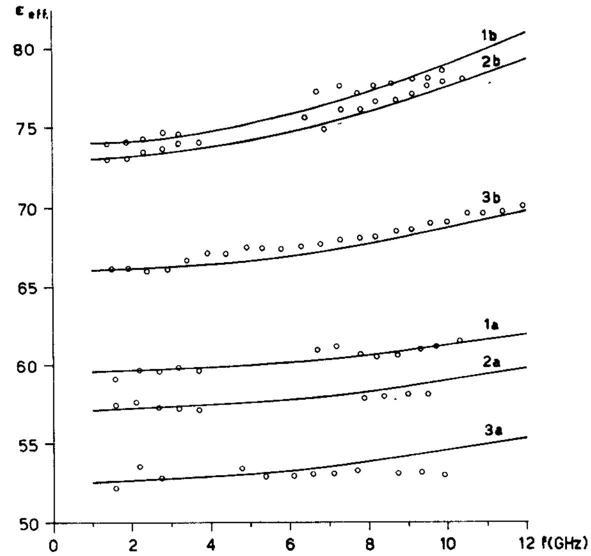

Fig. 2. Measured and calculated effective dielectric constant of TEM dispersive odd and even modes of parallel-coupled microstrips (see Table I for physical dimensions).

TABLE I
PARAMETERS OF PARALLEL-COUPLED MICROSTRIPS (AFTER GETSINGER [4])

Line	Mode	Static Impedance[a] Ω	Eff. Diel. Const.[b] at d.c.	W/H	S/H	H mm.
1a	odd	46.8	5.95			
1b	even	59.4	7.40	0.86	1.12	0.630
2a	odd	44.0	5.70			
2b	even	64.6	7.30	0.80	0.69	0.630
3a	odd	46.6	5.25			
3b	even	110.9	6.60	0.30	0.19	0.630

Notes: Column a calculated by the MSTRIP program using $\epsilon_s = 10.0$. Column b determined by extrapolation of experimental curves to zero frequency.

[1] These are not true TEM modes since they have a longitudinal H component, but they propagate to dc and for convenience are termed "dispersive TEM."

TABLE II
TE CUTOFF WAVENUMBERS OF PARALLEL-COUPLED MICROSTRIPS (ODD, TE_{01}; EVEN, TE_{10})

Line	Mode	$K_c = \pi/x$ (meters)$^{-1}$	x (mm.)	x/H
1a	odd	2066.6	1.52	2.41
1b	even	765.7	4.10	6.50
2a	odd	2061.6	1.52	2.41
2b	even	834.9	3.76	5.96
3a	odd	2233.1	1.41	2.24
3b	even	1391.9	2.26	3.60

The cutoff wavenumbers of the TE lines used in the model of the microstrips of Fig. 2 are reported in Table II. These numbers, K_{ce} and K_{co}, are parameters of the approximation and result from the assumption of using TE_{10} and TE_{01} as higher coupling modes. Thus they define the width and height of a hypothetical enclosing shield and, in effect, yield a plausible estimate of *equivalent shield* dimensions associated with the TE_{10}, TE_{01} cutoff wavenumbers. Thus from Tables I and II the equivalent height parameter (x/H, odd mode) is about 2.3 H for all geometries. The width parameter (x/H, even mode) varies from 6.5 H to 3.6 H (1.8/1), but note that S/H (S is conductor strip separation) varies over a 5.9/1 range. The substrate thickness H is 0.630 mm for all geometries.

In any case, just as in the dielectric loaded round guide [2], dielectric loaded rectangular guide [6], and single-strip microstrip [3], the coupled-line equivalent circuit gives a simple physical model with excellent experimental agreement for the dispersion properties of a parallel-coupled microstrip. We believe this further demonstrates that the coupled-line representation has broad applicability to a wide variety of longitudinally uniform, transversely inhomogeneous propagating structures.

REFERENCES

[1] S. A. Schelkunoff, "Generalized telegraphist equations for waveguides," *Bell Syst. Tech. J.*, vol. 31, pp. 784–801, July 1952.
[2] D. F. Noble and H. J. Carlin, "Circuit properties of coupled dispersive transmission lines," *IEEE Trans. Circuit Theory*, vol. CT-20, pp. 56–64, Jan. 1973.
[3] H. J. Carlin, "A simplified circuit model for microstrip," *IEEE Trans. Microwave Theory Tech.* (Short Papers), vol. MTT-21, pp. 589–591, Sept. 1973.
[4] W. G. Getsinger, "Dispersion of parallel-coupled microstrips," *IEEE Trans. Microwave Theory Tech.* (Short Papers), vol. MTT-21, pp. 144–145, Mar. 1973.
[5] G. W. Gould and E. C. Talboys, "Even- and odd-mode guide wavelengths of coupled lines in microstrip," *Electron. Lett.*, vol. 8, pp. 121–122, Mar. 9, 1972.
[6] H. J. Carlin, "Representation of dielectric loaded waveguide and microstrip by coupled line networks," in *Proc. 5th Colloquium Microwave Communication* (Budapest, Hungary, June 1974), vol. 3.

The Phase Shift Through Symmetrical 3-Port Circulators

G. RIBLET, MEMBER, IEEE

***Abstract*—Simple approximate formulas are derived for the phase shift through matched circulators—with and without transformer coupling—using expressions for the eigenadmittances Y_0, Y_{-1}, and Y_1 which have recently been proposed. These formulas allow one to predict the phase shift from measurements of the VSWR in one case and from a knowledge of the transformer admittance Y**

Manuscript received May 9, 1974; revised December 4, 1974.
The author is with the Microwave Development Laboratories Inc., Natick, Mass. 01760.